3) checkers:- (also called Reversi) 8x8 Square grid.
dev by Rosenbloom in 1982) Grid has
& different & light. Each face represent a player.

Goal:- make more peices of players. to have
of that colour face

4) Go 19x19 board.

each player uses Stones either black & white.
Goal is occupy larger part of the Board.

5) Back gamon:- Developed by Berlin's in 1980 playing
pices are moved by using Dice. player
must choose from neumerious Options.

UNIT-3 (Logic Concepts & programming)

1: Introduction

2. propositional calcus.

3. propositional logic

4. Natural deduction system.

5. Axiomatic System.

6. Sementic Tableau System in propositional logic

7. Resoultion Refutation in propositional logic

8. predicate logic.

INTRODUCTION

*) An effective way of Solving a complex probelem is To Reduce it to Simpler parts and Solve each part Separatly, This method is Called "problem - Reduction"

*) To Implement The above process, we need to use some logic.

(i) Logic is used to distinguish correct reasoning from Incorrect reasoning, which is concerned with The principle of drawing valid references from a given set of ~~TRUE~~ Two. Statements

(ii) Simbolic logic is divided into two types, They are:

1) propositional logic.

2) predicate logic.

propositional logic deals with a collection of declarative statements which have a truth value. [TRUE/FALSE]

predicate logic is an expression which consists of variables under a specific domain [its Truth value depends on variables]

A predicate with variables can be made a proposition by either Assigning a value to The variable or by ... the variable

Calculus is a method to Calculate reasoning.
reasoning is The process of applying logical and Critical Thinking to a probelem.

2.

2 PROPOSITIONAL CALCULUS

propositional Calculus (PC) is a language of propositions in which a set of rules are used to combine simple propositions to form compound propositions using some logical operators.

- The logical operators are called connectives, They are.

  $\wedge$ (AND), $\vee$ (OR), $\sim$ (NOT), $\rightarrow$ (implies), $\leftrightarrow$ (Equivalence)

- PC uses well formed formula, which is defined as symbol/string of symbols generated by formula Grammar of a formal Language.

- Some important properties of WFF in PC are,

  - The smallest unit/atom is considered to be WFF
  - if α is WFF, Then $\sim\alpha$ is also WFF
  - if α & β are WFF, then $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$, $(\alpha \rightarrow \beta)$, & $(\alpha \leftrightarrow \beta)$ are also WFF

- a propositional expression is called WFF if & only if it satisfies above properties

TRUTH Table

- A Truth table is used to provide operational definitions of important Logic operators
- The Logical constants in PC are represented as True(T) & False(F) in a TRUTH Table.

Eg:- Let us assume A, B as propositional symbols, Then Truth table is as follows $[(\sim A \vee B)]$, $[(A \wedge B) \vee (A \wedge \sim B)]$

A	B	$\sim A$	$A \wedge B$	$A \vee B$	$[\sim A \vee B]$ $A \to B$	$[A \wedge B] \vee (\sim A \wedge \sim B)$ $A \leftrightarrow B$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Equivalance Laws:-

used to reduce / Simply WFF (or) to derive new formula.

Name of Relation	Equalance Relation.
1) Commutative Law	$A \vee B \cong B \vee A$, $A \wedge B \cong B \wedge A$
2) Associative Law	$A \vee (B \vee C) \cong (A \vee B) \vee C$ $A \wedge (B \wedge C) \cong (A \wedge B) \wedge C$
3) Double Negation	$\sim(\sim A) \cong A$

) Distributive Law

$A \cup (B \cap C) \approx (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) \approx (A \cap B) \cup (A \cap C)$

) Demorgan's Law

$\sim (A \cup B) \approx \sim A \cap \sim B$

$\sim (A \cap B) \approx \sim A \vee \sim B.$

) Absorption Laws

$A \vee (A \cap B) \approx A$

$A \cap (A \cup B) \approx A$

$A \vee (\sim A \cap B) \approx A \cup B, \; A \cap (\sim A \vee B) \approx A \cap B.$

7) Idempotence Law

$A \vee A \approx A, \; A \cap A \approx A.$

8) Excluded Middle Law

$A \vee \sim A \approx T$ (True)

9) Contradiction Law

$A \cap \sim A \approx F$ (False)

10) Other Common Laws

$A \cup F \approx A, \; A \vee T \approx T$

$A \cap F \approx F, \; A \cap T \approx A$

$A \leftrightarrow B \approx (A \to B) \cap (B \to A)$

$\approx (A \cap B) \cup (\sim A \cap \sim B)$

3. PROPOSITIONAL LOGIC

- propositional Logic is used to check The validity, satisfiability, unsatisfibility of a formula using Equalance laws.

- A formula 'α' is said to be in 'Tautology', if the Value of 'α' is true for all interpretation.

<u>probelem</u>:- Prove That The below arguement is tautology

"if it is humid then it will rain and since it is humid today it will rain"

<u>Sol</u>:- First Symbolized The each part of above argument by using propositional atom

A: If it is humid

B: Then it will rain

The Sentence, "if it is humid Then it will rain and Since it is humid today it will rain"

Can be written as

$$\boxed{[(A \rightarrow B) \wedge A] \rightarrow B}$$

The Truth Table for 'α' is

A	B	$\sim A$	$(\sim A \vee B)$ $A \to B \approxeq x$	$x \wedge A \approxeq y$	$\sim y$	$\sim y \vee B$ $y \to B \approxeq \alpha$
T	T	F	T	T	F	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	F	T	T	F	T	T

Since 'α' is true for all interpretations, The Sentence is in Tautology.

4. NATURAL DEDUCTION SYSTEM

- A Natural deduction System Minimizes the pattern of natural reasoning.

- Natural deduction System is based on A set of deductive Inference Rules

- Assume That $A_1, A_2 \ldots A_k$ where $1 \le k \le n$ are set of atoms and α_j, j lies between $1 \le j \le n$ and β are well formed formulae, The Inference Rules are mentioned below.

Rule Name	Symbol	Rule	Description
Introducing $\wedge$	$(I: \wedge)$	if $A_1, \ldots A_n$ Then $A_1 \wedge \ldots \wedge A_n$	If $A_1 \ldots, A_n$ are true; Then Their Conjunction $A_1 \wedge \ldots \wedge A_n$ is also true.
Eliminating $\wedge$	$(E: \wedge)$	If $A_1 \wedge \ldots \wedge A_n$ Then A_i $(1 \le i \le n)$	if $A_1 \wedge \ldots \wedge A_n$ is true, Then any A_i is also true
Introducing $\vee$	$(I: \vee)$	If any $A_i (1 \le i \le n)$ Then $A_1 \vee \ldots \vee A_n$	if any A_i $(1 \le i \le n)$ is true, Then $A_1 \vee \ldots \vee A_n$ is also true.
Eliminating $\vee$	$(E: \vee)$	If $A_1 \vee \ldots \vee A_n$, $A_1 \to A$, $\ldots$ $A_n \to A$ Then A	if $A_1 \vee \ldots \vee A_n$, $A_1 \to A$, $A_2 \to A$, and $A_n \to A$ are true; Then A is true.
Introducing $\to$	$(I: \to)$	If from $\alpha_1, \ldots \alpha_n$ infer β is proved $\alpha_1 \wedge \ldots \wedge \alpha_n \to \beta$ is proved	if given That $\alpha_1, \alpha_2, \ldots$ and α_n are true and from Those we deduce β Then $\alpha_1 \wedge \ldots \wedge \alpha_n \to \beta$ is also true.

Eliminating $\rightarrow$	$(E:\rightarrow)$	if $A_1 \rightarrow A, A_1$, Then A	if $A_1 \rightarrow A$ and A_1 are true Then A is also true. This is Called modes ponen Rule.
Introducing $\leftrightarrow$	$(I:\leftrightarrow)$	if $A_1 \rightarrow A_2, A_2 \rightarrow A_1$ Then $A_1 \leftrightarrow A_2$	if $A_1 \rightarrow A_2$ and $A_2 \rightarrow A_1$ are true Then $A_1 \leftrightarrow A_2$ is also true
Eliminating $\leftrightarrow$	$(E:\leftrightarrow)$	if $A_1 \leftrightarrow A_2$ Then $A_1 \rightarrow A_2$, $A_2 \rightarrow A_1$	if $A_1 \leftrightarrow A_2$ is true. Then $A_1 \rightarrow A_2$ and $A_2 \rightarrow A_1$ are true
Introducing $\sim$	$(I:\sim)$	if from A infer $A_1 \wedge \sim A_1$ is proved Then $\sim A$ is proved	If from A (which is true), a Contradiction is proved Then truth of $\sim A$ is also proved.
Eliminating $\sim$	$(E:\sim)$	if from $\sim A$ infer $A_1 \wedge \sim A_1$ is proved Then A is proved	if from $\sim A$, a Contradiction is proved Then truth of A is also proved.

problem:- using NDS. (Natural deduction System) prove That $A \wedge (B \vee C)$ is deduted from $A \wedge B$.

Ans:- The above statement can be written in NDS as

from $A \wedge B$, infer $A \wedge (B \vee C)$.

	Description	formula	Comment
1.	Theorem	from $A \wedge B$, infer $A \wedge (B \vee C)$	To be proved
2.	Hypothesis	$A \wedge B$	1
3.	$E: \wedge$	A	2
4.	$E: \wedge (1)$	B	3
5.	$I: \vee (3)$	$B \vee C$	4
6.	$I: \wedge (2,4)$	$A \wedge (B \vee C)$	

5. AXIOMATIC System

*) An Axiom is a statement and(or) proposition which is true.

*) The Axiomatic System is Based On a set of 3-axioms and one rule of deduction

*) Here ~(not) and → (implies) are only allowed to form a formula. ie: $\wedge$, $\vee$ & $\leftrightarrow$ are to be expressed only By using ~ and →

Ex:-

1) $A \wedge B \approxeq \sim(\sim A \vee \sim B)$

$\approxeq \sim(A \to \sim B)$

2) $A \vee B \approxeq \sim A \to B$

3) $A \leftrightarrow B \approxeq (A \to B) \wedge (B \to A)$

$\approxeq \sim[(A \to B) \to \sim(B \to A)]$

*) In axiomatic System There are 3 axioms which are always true with rule Modus (Ponen's Rule)

Here α, β and γ are well formed formula of The axiomatic System.

*) The 3 axioms and one rule of axiomatic System as

Axiom 1: $\alpha \rightarrow (\beta \rightarrow \alpha)$

Axiom 2: $[\alpha \rightarrow (\beta \rightarrow \gamma)] \rightarrow [(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)]$

Axiom 3: $(\sim\alpha \rightarrow \sim\beta) \rightarrow (\beta \rightarrow \alpha)$

1 Rule.

Modus ponem : Hypothesis : $\alpha \rightarrow \beta$ and α.

Consequent: β

* The modus ponem's rule says that, given that $\alpha \rightarrow \beta$ and α are hypothesis (Assume to be true), β is inferred (true) as a consequent

Problem :-

Using axiomatic system, establish that $A \rightarrow C$ is a deductive consequent ($\vdash$) of $\{A \rightarrow B, B \rightarrow C\}$ i.e.

$$\{A \rightarrow B, B \rightarrow C\} \vdash A \rightarrow C$$

{$\vdash$ deduction symbol}

Solution:-

Prove that theorem that $\{A \rightarrow B, B \rightarrow C\} \vdash A \rightarrow B$

Description	Formula	Comment
Theorem	$(A \to B, B \to C) \vdash A \to C$	To be proved.
Hypon Thesis 1	$A \to B$	1
Hypothesis 2	$B \to C$	2
Instance of Axiom 1	$B \to C \to [A \to (B \to C)]$	3
Modus ponem's (2,3)	$A \to (B \to C)$	4
Instance of Axiom 2	$[[A \to (B \to C)] \to [(A \to B) \to (A \to C)]]$	5
modus ponem's (4,5)	$(A \to B) \to (A \to C)$	6
modoss ponen's (1,6)	$A \to C$	Hence proved

6. Semantic Tableau System In propositional Logic

* Both 'Natural Deduction' System' & 'Axiomatic System' uses 'forward changing approach' for Constructing proofs and derivations.

*) The other two approaches which follows 'Backward changing' are.

- Semantic Tableau
- Resolution Refutation methods.

*) In Semantic tableau method, a set of rules are applied systematically in a formula & a set of formulae in order to establish consisting (or) inconsistency

*) Semantic tableau is a binary tree. which is constructed by using Semantic tableau rules with a formulae as a root

Rule No.	Tableau Tree.
Rule 1	$\alpha \wedge \beta$ | α | β
Rule 2	$\sim(\alpha \wedge \beta)$ $\sim\alpha$ $\sim\beta$

Rule

Rule

Rule

Rule

Rule

Rule 8

Rule 9

Ty

p

Rule 3	$\begin{array}{c} \alpha \lor \beta \\ \swarrow \quad \searrow \\ \alpha \qquad \beta \end{array}$
Rule 4	$\begin{array}{c} \sim(\alpha \lor \beta) \\ \mid \\ \sim\alpha \\ \mid \\ \sim\beta \end{array}$
Rule 5	$\begin{array}{c} \sim(-\alpha) \\ \mid \\ \alpha \end{array}$
Rule 6	$\begin{array}{c} \alpha \rightarrow \beta \\ \swarrow \quad \searrow \\ -\alpha \qquad \beta \end{array}$
Rule 7	$\begin{array}{c} \sim(\alpha \rightarrow \beta) \\ \mid \\ \alpha \\ \mid \\ \sim\beta \end{array}$
Rule 8	$\begin{array}{c} \alpha \leftrightarrow \beta \\ \swarrow \quad \searrow \\ \alpha \land \beta \qquad \sim\alpha \land \sim\beta \end{array}$
Rule 9	$\begin{array}{c} \sim(\alpha \leftrightarrow \beta) \\ \swarrow \quad \searrow \\ \alpha \land \sim\beta \qquad \sim\alpha \land \beta \end{array}$

Types of Semantic tableau :-

problems :-

1. Construct a Semantic tableau from formula.
2. Show That a formula is unsatisfiable/ satisfible using tableau Section

- Show That a formula is Consistent
- Show That as axiom is a logical Consequence of formula.
- Show That a formula is Valid.

- Constrat a Semantic tableau for a formula.

$(A \wedge \sim B) \wedge (\sim B \rightarrow C)$

Rule applied	Formula	Line number.
Tableau root	$(A \wedge \sim B) \wedge (\sim B \rightarrow C)$	1
Rule 2 (1)	$A \wedge \sim B$	2
	$\sim B \rightarrow C$	3
Rule 1 (2)	A	4
	$\sim B$	5
	Left branch: $\sim(\sim B)$, B × (Closed) Since, we have Complementary atoms $\{B, -B\}$ Right branch: C (open) Since no Complementary atoms	6

7. Resolution Refutation in propositional Logic

i) To prove a formula or derive a goal from a given set of clauses by contradiction is done by "Resolution Refutation" method.

*) clause is used to denote special formula containing operations ~ and v.

*) Any formula can be converted to set of clauses

*) RR uses single inference rules, which is known as resolution based on 'modus ponen' inference rule.

*) During resolution, we need to identify 2 clauses. One with positive atom (P) and other with negative atom (~P)

Conversion of a Formula into a set of clauses :-

*) A function is said to be in normal form if it is constructed using only natural connectives (∧, v, ~)

*) Normal forms can be DNF (disjunction) or CNF (conjunction)

*) CNF is a conjunction of disjunction where as DNF is a disjunction of conjunction as

to its CNF.

Ex:

Ans:

Rules to convert formula

- $\sim(\sim A) \approx A$
- $\sim(A \wedge B) \approx \sim A \vee \sim B$, $\sim(A \vee B) \approx \sim A \wedge \sim B$
- $A \vee (B \wedge C) \approx (A \vee B) \wedge (A \vee C)$
- $A \to B \approx \sim A \vee B$
- $A \leftrightarrow B \approx (A \to B) \wedge (B \to A)$

problem:-
Convert The probelem formula $(\sim A \to B) \wedge (C \wedge \sim A)$ into CNF

Sol:-

$$(\sim A \to B) \wedge (C \wedge \sim A) \approx (\sim(\sim A) \vee B) \wedge (C \wedge \sim A)$$
$$\approx (A \vee B) \wedge (C \wedge \sim A)$$
$$\approx \underbrace{(A \vee B)}_{\text{Clauses 1}} \wedge \underbrace{C}_{\text{Clauses 2}} \wedge \underbrace{\sim A}_{\text{Clauses 3.}}$$

The Set of Clauses are. $A \vee B$, C, $\sim A$ which Can be written as

$$\{(A \vee B), C, \sim A\}.$$

Resolution of clauses:- Two clauses can be resolved by eliminating completing pair of literals, from a new clauses is constructed by division of remaining literals on both with

Ex:- find The resolvent of The clauses in The set $\{A \vee B, \sim A \vee D, C \vee \sim B\}$

Ans:

$A \vee B$

$\sim A \vee D$

$(A, \sim A)$

$B \vee D$

$C \vee \sim B$

$(B, \sim B)$

$C \vee D$ (Resolvent)

from The figure, $C \vee D$ is Resolvent

8. predicate Logic

- A predicate is an expression of one or more variables determined on a specific domain (i.e. we store new knowledge or infer new knowledge.

- A predicate with variables can be made a proposition by authorizing a value to a variable or by quantifying a variable.

- There are 2 types of quantifier in predicate logic.

  - Existential quantifier, denoted by "$\exists$" (There exist)
  - Universal quantifier, denoted by "$\forall$" (for all)

Eg:-

Existential Quantifier:- ($\exists$)

- we use "$\wedge$" (conjunction) / AND
- if $p(x)$ is a proposition over universe 'U' is denoted as $\exists x\ p(x)$ (There exists at least one value in the universe of variable x such that $p(x)$ true)

Universal Quantifier ($\forall$)

- If $p(x)$ is a proposition over the universe 'U' is denoted as $\forall x, p(x)$ [for every $x \in U$, $p(x)$ is true)

First - order - predicate (FOPL) Logic / predicate Logic

*) FOPL is another extension of propositional logic which is more efficient than above / previous mechanisms

FOPL = { predicate Calculus inference rules }.

*) FOPL not only assumes facts but also assumes things like.

- objects
- Relations
- functions

*) FOPL has 2 main parts

- Syntax
- Semantics.

*) Syntaxs in FOPL is a allocation of symbols in a logical expression. These symbols are also called as "Syntactic elements".

*) The various categeories of Synthetic elements / Symbols are.

- Variables x, y, z
- Constant - india, deepa, Aadhar no.
- predicates father, mother, Brother, Sister.
- function power, Sort
- Connectiones $\wedge, \vee, \sim, \rightarrow, \leftrightarrow$
- Equality ==
- Quntifier $\exists, \forall$

* Sentences in FOPL are of 2 types
  - Atomic Sentence
  - Complex sentence.

*) Atomic Sentences are basic Sentences which are formed from a predicate Symbol followed paranthesis with a Sequence of terms

Syntax! - predicate Symbol ($term_1$, $term_2$, ... $term_n$)

Ex :-

1. Ravi & Ajay are brothers can be written as Brothers (Ravi, ajay)

2. Catty is a cat
Cat (Catty)

x) Complex sentences are formed by combining atomic sentences using connectives

* FOPL statements can be divided into 2 parts

- Subject (main part)
- predicate (relation which binds 2 atoms together)

Eg:-

x (Subject)

is an integer (predicate)

Subject & predicate :- x is $\underbrace{\text{an integer}}$

$\downarrow$ Sub. $\downarrow$ predicate

FOPL :-

Quntifier Eg's :-   Symbol   Connectives.

1. Existential :- ($\exists$) , '$\wedge$'

" Some students are Intelligence"

$\exists x$:- students (x) $\wedge$ intelligence (x)

There are some x where x is a student who is

2. Universal :- ($\forall$)

" All students drinks milk "

$\forall x$ students $(x) \longrightarrow$ drink$(x, milk)$

for all x where x is a student will drink milk.

Note :-

Above students cannot be represented in propositional logic, which is over come by predicate logic.